# Trigonometry Review For High School Physics

Trigonometry is used in physics most extensively in the application of vectors to describe locations, speeds, and forces. Trigonometry is a collection of definitions, relationships and facts that make an understanding of physics more complete. If you have not yet had a course in trigonometry, or if you wish to have a review of what you already know, this document will provide you with the skills and background you need to succeed in physics.

Definitions of the Trigonometric functions:

There are 6 trigonometric functions that are defined as the six possible ratios of sides in a right triangle. They are named sine, cosine, tangent, cosecant, secant and cotangent: sin, cos, tan, csc, sec and cot. Only the first three of these functions are commonly used in physics, so we will restrict our discussion to these three functions. (The other three are simply the reciprocals of these three anyway, which is why you’ll never see a calculator with csc, sec or cot.)

When defined on a right triangle such as the one shown here, sin, cos and tan are defined as the ratio of two sides of the triangle. The side opposite the right angle is called the **hypotenuse (**hyp), the side that, along with the hypotenuse, makes the angle , is called the **adjacent**(adj) side, and the third side is called the **opposite** (opp) side because it is directly opposite the angle . The sin, cos and tan are defined as follows.



hypotenuse

opposite

adjacent





Notice that the orientation of the angle and triangle may differ, but the definitions of the three functions are always the same relative to that angle and triangle.

adj

opp

hyp

adj

hyp

adj

adj

opp

opp

hyp

hyp

opp

Exercises: Consider the triangle drawn and identify the value of each trig function indicated.

1. sin A= sin B=

B

A

3

4

5

 cos A= cos B=

 tan A= tan B=

13

2.

D

5

C

12

sin C= sin D=

 cos C= cos D=

 tan C= tan D=

7

3.

F

sin E= sin F=

 cos E= cos F=

 tan E= tan F=

25

24

E

All About Angles:

As with any function, the trigonometric functions have a domain and a range. The domain consists of angles, while the range of the three functions are real numbers. You can think of a function as a black box. An angle is fed into the trigonometric function, the function operates on this angle and gives a number as the value of the function at this angle. The input to a function is called the argument. The argument of the trig functions is always an angle.

angle

value

Trig function

Angles are indicated in one of two ways: in radians or in degrees. A radian is an arc length equal to the radius of the circle. The circumference of a circle is always 2π radians and 360°. The diagram below shows the identity of the “special angles” in both radians and degrees.

0° 0

360° 2π

30° π/6

45° π/4

60° π/3

π/2

90°

2π/3 120°

3π/4 135°

5π/6 150°

π 180°

7π/6 210°

5π/4 225°

4π/3 240°

270° 3π/2

300° 5π/3

315° 7π/4

330° 11π/6

Angles are measured from the positive x axis and wrap around a circle in the counter clockwise direction. If you continue to wrap around the circle, the measure of the angle can become more than 360° or less than 0°. In fact, angles can have any real number value, in either degree or radian notation. A negative angle means an angle measured in a clockwise direction.

These angles are “special” because they are related to the two “special” triangles in geometry: the 45-45-90 triangle and the 30-60-90 triangle. The sides of these two triangles have integer or simple square root lengths:

60°

45°



2

1

1

30°

45°



1

Another definition of sinθ and cosθ:

Imagine a circle with a radius of one unit placed at the origin of a Cartesian coordinate plane. Every point on this circle has an *x* and a *y* coordinate. The special triangles above can help derive the values of the coordinates for each point on the unit circle that intersect with the special angles.

































The *x* coordinate of every point on the unit circle is equal to the cosine of the angle that intersects that point. The *y* coordinate of every point on the unit circle is equal to the sine of the angle that intersects that point. So each of the coordinates above can be expressed as . The tangent of that angle is equal to the *y* coordinate divided by the *x* coordinate.

For a circle of any other size, the sine and cosine can be defined in terms of the *x* and *y* and the radius, *r*:

  

The first two of these relationships can be rewritten:





## Sign Patterns for sinθ, cosθ and tanθ

Because sinθ is correlated with *y*, the value of sinθ will have the same sign as the *y* coordinate of each quadrant. Because cosθ is correlated with *x*, the value of cosθ will have the same sign as the *x* coordinate of each quadrant. The sign of tanθ will be positive if the *x* and *y* coordinates are the same sign, and will be negative it they have different signs.

sinθ cosθ tanθ

+

–

 –

 +

+

–

 +

 –

+

+

 –

 –

Reference Angles

For every angle θ in the first quadrant there is a corresponding angle in each of the other three quadrants with the same absolute values for sinθ, cosθ and tanθ, differing only by the sign patterns indicated above. These corresponding angles are called reference angles because they refer to the angle θ. They can be found as shown below.

180° –

180° +

360° –

## Graphs of sinθ, cosθ and tanθ

These functions can be graphed with the *x*-axis for the angle and the *y*-axis for the value of the function. From these graphs, one can see why these functions are often called periodic functions. The graphs repeat after each revolution around the circle. Below are graphs of sinθ, cosθ, and tanθ from –4π (–720°) to 4π ( 720°) to show the repetition.

Sin θ: Notice that the range of sinθ is between –1 and 1.

Cosineθ: Notice that the shape of sine and cosine are identical, expect that cosine starts at 1 when the angle is 0, while sine starts at 0 when the angle is 0.

Tanθ: Notice that the repeat unit for tangent is half the size of the sine and cosine.

Inverse Trigonometric functions

An inverse function takes the output of a function as its input, and gives as its output the input of the function. It’s a function box working backwards.

angle

value

Inverse

Trig function

The notation of an inverse trig function is to place a –1 “exponent” in between the trig function and its argument. E.g. sin-1θ. The argument for an inverse function is the *value* of the corresponding trig function. The output of an inverse trig function is an *angle*.

The domains of the inverse functions are the ranges of the regular trig functions. For example: The range of Sinθ is between –1 and 1. Therefore the domain for sin-1θ is between –1 and 1.

The range of inverse functions are NOT equivalent to the domain of the regular trig functions. The reason for this is due to the periodic nature of the trig functions. For example: Consider a sinθ value of 0.5. There are many points on the graph of sinθ that have a y value of 0.5. In order to be a function (that is, to have only a single possible output), the domain of the inverse must be restricted to a particular interval of angles.

The inverse functions for sinθ, cosθ, and tanθ, by definition, have the following ranges:

Sin-1θ.lies between –90° and +90°

Cos-1θ lies between 0 and 180°

Tan-1θ lies between –90° and +90°

Notice that each of these ranges covers both a positive quadrant and a negative quadrant for each function. This produces a one to one correspondence between the value of a trig function and a set of angles.

A CALCULATOR WILL ONLY GIVE YOU AN ANGLE WITHIN THESE RANGES!

But sometimes you need to know the other angle that has the same function value. This commonly occurs when evaluating inverse tangent functions. The rule to use with inverse tangent is if the *x* value for the ratio of *y/x* is negative, then to obtain the correct angle you must add 180° to the result your calculator provides. It is possible to find a second angle for sine and cosine as well, but is less common. If you need to do this, the second angle is the reference angle in the other quadrant with the same sign. E.g. Second angle for a positive sine value is lies in the second quadrant and is found using 180° – θ

Using your calculator to evaluate trig and inverse trig functions:

First, verify that your calculator is in the proper mode. This is usually the degree mode, but you will need to use radians when describing oscillations and rotations, so keep that in mind.

If you have a graphing calculator, you will evaluate trig functions by pressing the appropriate trig button, followed by the angle and “)”. To evaluate an inverse trig function, press 2nd then the appropriate trig button, followed by the value of the trig function, and “)”. If you have the angle, use the trig function. If you want to find the angle, use the inverse trig function.

If you have a scientific calculator, you will evaluate trig functions by entering the angle followed by pressing the appropriate trig button. To evaluate an inverse trig function, enter the value of the function, then press 2nd followed by the trig button.

Exercises:

For the three triangles presented earlier, determine the measure of the angles A, B, C, D, E and F.